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STUDENTS' THINKING PROCESSES IN INVERSE PROPORTION PROBLEM SOLVING

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ABSTRACT
This study describes students' thinking processes in solving inverse proportion problems based on mathematical ability. The study employed a qualitative descriptive design involving three students categorized as having high, moderate, and low mathematical ability. Data were collected through a written inverse proportion task and task-based interviews organized according to four problem-solving stages: understanding the problem, devising a plan, carrying out the plan, and looking back. The task asked students to determine the completion time when 20 workers who could finish a job in 30 days were increased to 50 workers. The results showed that the high-ability student recognized the inverse relation, used the constant-product model $20 \times 30 = 50 \times x$, obtained 12 days, and verified the answer. The moderate-ability student treated the situation as direct proportion by writing $20 : 50 = 30 : x$, indicating relational misconception. The low-ability student recognized the inverse relation but had difficulty with multiplication and division and did not check the answer. The findings imply that inverse proportion instruction should emphasize contextual relations, strategy selection, computational fluency, and reflective checking.

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1. INTRODUCTION

Mathematics learning is closely related to students' thinking processes because mathematical activity requires learners to interpret information, organize relations, select strategies, and justify conclusions. Word problems are a productive context for observing these processes because students must connect verbal information with mathematical structure before applying procedures. They need to decide which quantities are relevant, how quantities are related, and whether the selected operation remains reasonable in context. Studies of mathematical word problems therefore emphasize that success depends on comprehension, representation, and quantitative reasoning rather than calculation alone (Boonen et al. 2016; Daroczy et al. 2016; Pongsakdi et al. 2020; Verschaffel et al. 2020).

Proportion reveals relational reasoning because students must coordinate two quantities and identify how changes in one quantity correspond to changes in another. In direct proportion, quantities move in the same direction, whereas in inverse proportion an increase in one quantity is accompanied by a decrease in the other. Although this distinction seems simple, it is demanding because students often focus on surface numbers before interpreting the embedded relation. Research on ratio, proportion, and quantitative literacy shows that learners may have difficulty interpreting multiplication and division as relational operations in contextual forms (Boudreaux, Kanim, and Brahmia, 2016; Smith et al. 2020; Brahmia et al. 2021).

The worker-time problem used in this study illustrates this demand. The task states that 20 workers can complete a job in 30 days and asks the days required if the workers become 50. The key idea is that total work remains constant; therefore, the product of workers and days must remain constant, so the expected model is $20 \times 30 = 50 \times x$. A student who interprets the situation correctly should anticipate that the answer must be less than 30 days.

Students often experience difficulty because familiar procedures dominate contextual interpretation. Cross-multiplication is commonly taught for finding missing values in proportion tasks. If applied mechanically, students may construct a direct proportion even when the situation requires inverse proportion. Such errors are not merely computational; they reflect a mismatch between the real situation and the mathematical model. Mathematical modelling and word-problem research show that students need to move from situation to representation and then return to the situation to test whether the result is plausible (Schukajlow, Kaiser, and Stillman 2018; Krawitz, Schukajlow, and Van Dooren 2018; Verschaffel et al. 2020).

The distinction between direct and inverse proportion also requires students to recognize whether a task involves a constant ratio, product, sum, or difference. When students overgeneralize direct proportion, they may assume that two quantities increase together simply because one quantity increases. Krawitz et al. (2018) discussed students' adaptive but sometimes overextended use of linearity, while Brahmia et al. (2021) emphasized covariational reasoning, namely attending to how one quantity changes as another changes. These perspectives are central because workers and time change in opposite directions while total work remains invariant.

Word problems contain linguistic and numerical features that increase difficulty. Daroczy et al. (2016) showed that language, numerical information, and semantic structure contribute to word-problem complexity. In a worker-time task, "more workers" may draw attention to increase, whereas the intended relation requires students to infer that time decreases. This explains why students may identify the numbers correctly but still choose an inappropriate operation. Enriched word problems and discussion of contextual meaning can help students overcome this tendency (Pongsakdi et al. 2020). Schema-based instruction can support proportional thinking by helping students identify the underlying structure before selecting a procedure (Jitendra et al. 2016), while pre-algebraic reasoning strengthens students' ability to represent relations in word problems (Daroczy et al. 2016).

Problem-solving frameworks are useful for identifying where students' thinking succeeds or fails. This study uses four stages commonly associated with Polya's framework: understanding the problem, devising a plan, carrying out the plan, and looking back. These stages structure how students identify known and unknown quantities, select strategies, perform procedures, and check whether answers are reasonable. Staged problem solving helps distinguish comprehension, planning, implementation, and verification (Liljedahl et al. 2016; Novotná et al. 2019; Rott, Specht, and Knipping 2021). In inverse proportion, students may fail at different points: selecting the relation, computing, or checking the answer.

Problem solving also requires monitoring. (Hidayat, Zulnaidi, and Zamri, 2018) showed that metacognitive processes are related to mathematical modelling competence. Mason and Singh (2016) demonstrated the value of reflection, and Phillips et al. (2016) introduced a framework emphasizing activation, construction, execution, and reflection in STEM tasks. In inverse proportion, monitoring appears when students ask whether a result is consistent with the condition that more workers should require fewer days. Huang et al. (2016) also emphasized that mathematical problem solving involves resources, heuristics, control, and sense making. Thus, success should be judged not only from the final answer, but also from understanding, strategy, control, and evaluation.

The issue becomes more complex when students have different mathematical abilities. Mathematical ability affects conceptual understanding, procedural fluency, strategic competence, and self-monitoring. Ayuningtyas, Mardiyana, and Pramudya (2019) showed that students' problem-solving skills can be examined in relation to mathematical ability because different levels may produce different patterns of understanding and strategy use.

Conceptual and procedural knowledge are both necessary for solving inverse proportion problems. Rittle-Johnson (2017) explained that mathematical knowledge develops through the interaction between conceptual and procedural knowledge. Recognizing an inverse relation does not automatically guarantee that a student can represent and calculate the missing value accurately. Boudreaux et al. (2016) emphasized that understanding rational number relations is central to broader mathematics achievement, while Fuchs et al. (2021) showed that students' mathematical difficulties may involve understanding, calculation, and word-problem performance simultaneously.

Verification is an important marker of reflective problem solving. Students should not stop after obtaining a number; they should examine whether the answer fits the situation. Krawitz et al. (2018) showed that students may give unrealistic responses to realistic problems when they do not evaluate contextual constraints. Huang et al. (2016) also showed that instructional support can reduce mathematics performance gaps, suggesting that guidance in checking and interpreting solutions may support students who have difficulty monitoring their work. This study therefore treats looking back as an essential stage rather than an optional final step.

Although many studies have addressed word problems, proportional reasoning, mathematical modelling, and problem solving, descriptions of how students with different mathematical abilities move through the same inverse proportion task remain useful for classroom diagnosis. A focused qualitative analysis can reveal whether an error arises from incomplete comprehension, relational misconception, procedural difficulty, or weak verification. Therefore, this study aims to describe students' thinking processes in solving inverse proportion problems based on mathematical ability. It compares three students categorized as high, moderate, and low through task-based interviews organized by the four problem-solving stages.

2. METHOD

This study employed a qualitative descriptive design to examine students' thinking processes in solving an inverse proportion problem. The design was appropriate because the study aimed to describe how students understood the task, selected strategies, performed calculations, and checked answers rather than to measure achievement statistically. Qualitative educational research is suitable for exploring participants' reasoning, meaning-making, and explanations in context (Johnson, Adkins, and Chauvin 2020; Levitt et al. 2018).

The study involved three students categorized by mathematical ability: S1 represented high ability, S2 moderate ability, and S3 low ability. Their identities were anonymized using subject codes. Although the number of participants was limited, the selection provided contrasting information, consistent with qualitative adequacy based on information power, specificity, and richness of data (Schukajlow et al. 2018).

The main instrument was an inverse proportion task involving 20 workers, 30 days, and 50 workers, with the expected model $20 \times 30 = 50 \times x$. Task-based semi-structured interviews explored understanding, planning, implementation, and verification (Adeoye-Olatunde and Olenik 2021; Cypress 2017). Data were analyzed by classifying responses into these stages and comparing completeness, appropriateness, procedural accuracy, and verification across subjects.

3. RESULT AND DISCUSSION

Results

The results are presented according to the thinking processes of the three subjects. The analysis follows the four problem-solving stages: understanding the problem, devising a plan, carrying out the plan, and looking back. The comparison shows that the three students differed not mainly in reading the numbers in the problem, but in interpreting the mathematical relationship, selecting the appropriate procedure, executing the calculation, and verifying the answer.

Thinking Process of S1: High Mathematical Ability

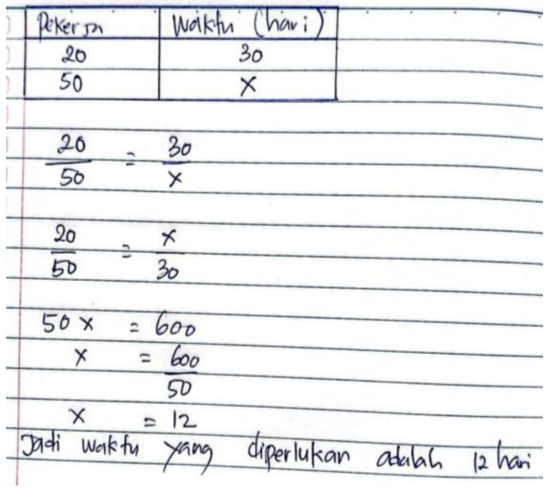

Figure 1. Subject 1's Answer Sheet

Understanding the Problem

At the stage of understanding the problem, S1 identified all essential information correctly. The student stated that 20 workers could complete the work in 30 days and that the question asked for the time required if the number of workers became 50. S1 also explained the relation between the quantities by stating that when the number of workers increases, the work can be finished faster, so the required time decreases. This response shows that S1 did not only read the numbers but also interpreted the real-world relationship between workers and duration.

Devising a Plan

At the planning stage, S1 selected inverse proportion as the appropriate method. The justification was based on the opposite movement of the two quantities: the number of workers increased while the time decreased. S1 also rejected direct proportion because it would produce an unreasonable result; if the number of workers increases, the time should not increase. This reasoning shows that S1 considered contextual meaning before applying a mathematical procedure. Such behavior is consistent with the view that successful word-problem solving requires the construction of a meaningful mathematical relation (Boonen et al. 2016; Verschaffel et al. 2020).

Carrying Out the Plan

At the implementation stage, S1 wrote the equation $20 \times 30 = 50 \times x$. The student then calculated $x = 600/50$ and obtained $x = 12$ days. The procedure represents the constant-product structure of inverse proportion, where the total work remains unchanged although the number of workers changes. This solution shows coordination between conceptual understanding and procedural skill. The connection between conceptual and procedural knowledge is important because correct reasoning must be accompanied by accurate calculation (Rittle-Johnson, 2017).

Looking Back

At the verification stage, S1 checked whether the answer was reasonable. The student compared the new condition with the original condition and concluded that 50 workers were more than 20 workers, so the required time had to be less than 30 days. Because 12 days was less than 30 days, S1 considered the answer reasonable. This stage shows reflective thinking and metacognitive control. The student did not merely trust the calculation but interpreted the result in relation to the original problem situation.

Thinking Process of S2: Moderate Mathematical Ability

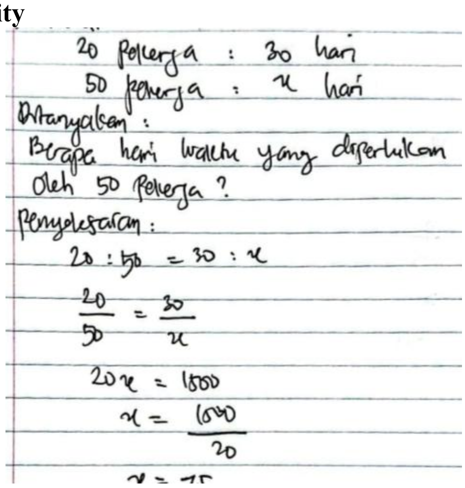

Figure 2. Subject 2's Answer Sheet

Understanding the Problem

At the understanding stage, S2 identified the given information accurately. The student stated that there were 20 workers who completed the work in 30 days and that the number of workers became 50. S2 also stated that the question asked for the length of time required. This indicates that S2's difficulty did not occur in recognizing the numbers or the unknown quantity. The difficulty appeared when the student had to determine how the quantities were related.

Determining the Relationship Between Quantities

When asked about the relation between the number of workers and time, S2 showed uncertainty. The student initially thought that because the number of workers increased, the situation could be solved using an ordinary proportion. This response shows a misconception in distinguishing direct and inverse proportion. The student focused on a familiar proportional procedure rather than on the contextual meaning of the worker-time relation. This result supports the view that word-problem errors often occur when students rely on surface numerical cues instead of the underlying mathematical structure (Daroczy et al. 2016; Verschaffel et al. 2020).

Devising a Plan

At the planning stage, S2 chose cross-multiplication because it was familiar from proportion problems. However, S2 did not first examine the contextual relationship between workers and time. The student overlooked that an increase in workers should reduce the completion time. This shows that procedural habit dominated relational reasoning.

Carrying Out the Plan

At the implementation stage, S2 wrote 20 : 50 = 30 : x and then performed cross-multiplication. This procedure is inappropriate because it treats workers and time as if they increase or decrease together. The expression does not preserve the constant amount of work and therefore does not model the inverse relationship. The error illustrates that symbolic procedures may be mathematically invalid when they are not guided by conceptual interpretation. This finding aligns with the argument that mathematical modelling requires movement from context to mathematics and back to context (Schukajlow et al., 2018).

Looking Back

At the verification stage, S2 did not check the answer independently. After the researcher asked whether the time should increase or decrease when the number of workers increases, S2 acknowledged that the time should decrease. The student then recognized that the previous answer might be incorrect because the relationship between workers and time had not been properly considered. This prompted recognition suggests that S2 had partial contextual awareness but did not activate it during the initial solution process. The case shows the importance of explicit verification prompts in mathematics instruction.

Thinking Process of S3: Low Mathematical Ability

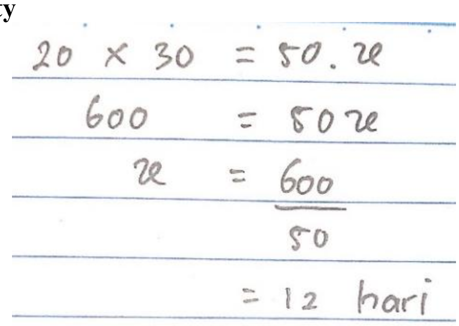

Figure 3. Subject 3's Answer Sheet

Understanding the Problem

At the understanding stage, S3 identified the main quantities in the problem. The student stated that the problem involved 20 workers, 30 days, and a change to 50 workers. S3 also recognized that the question asked for the number of days required. Although the explanation was shorter than S1's explanation, the essential information was mostly identified. This shows that S3 was able to access the basic meaning of the problem statement.

Determining the Relationship Between Quantities

At the relation-identification stage, S3 stated that when the number of workers increases, the required time decreases. The student correctly classified the problem as inverse proportion and justified the choice by explaining that more workers make the work finish faster. This response shows that low mathematical ability in this case did not prevent the student from recognizing the appropriate concept. The result confirms that conceptual recognition and procedural performance should be analyzed separately.

Devising a Plan

At the planning stage, S3 chose inverse proportion because the number of workers increased and the work should be completed more quickly. The plan was conceptually appropriate, but the student showed limited confidence in transforming the idea into a stable symbolic procedure. This pattern indicates that a correct plan at the conceptual level does not automatically ensure successful implementation. Students may require additional support to connect verbal reasoning, symbolic representation, and arithmetic execution.

Carrying Out the Plan

At the implementation stage, S3 attempted to write the inverse proportion formula but became confused when multiplying and dividing the numbers. When asked which part was difficult, S3 stated that the difficulty occurred during the calculation. Thus, S3's main obstacle was procedural or computational rather than conceptual. This supports the argument that conceptual and procedural knowledge are related but not identical (Rittle-Johnson, 2017). A student may understand why the relationship is inverse but still fail to produce a correct numerical answer.

Looking Back

At the verification stage, S3 did not check the answer because the student was already uncertain about the calculation. This response indicates that computational insecurity can prevent students from engaging in reflection. When students are not confident in their procedures, they may stop before evaluating whether the result is reasonable. Therefore, the looking-back stage requires not only metacognitive awareness but also sufficient confidence and fluency to compare the result with the context.

Cross-Subject Summary of Findings

Across the three subjects, all students identified the basic numerical information in the problem, but their thinking differed after the initial comprehension stage. S1 demonstrated complete problem solving by recognizing the inverse relationship, selecting an appropriate strategy, carrying out the calculation accurately, and independently verifying the answer. S2 understood the known and asked information but misinterpreted the relationship between workers and time, leading to the use of a direct proportion procedure. S3 recognized the inverse relationship but lacked confidence in carrying out the calculation, so the solution was not completed accurately. Only S1 checked whether the answer was reasonable. These findings indicate that mathematical ability influenced reasoning quality beyond information recognition. Errors in inverse proportion may occur at different stages: relational interpretation, symbolic representation, computation, or verification. Therefore, each type of difficulty requires different instructional support.

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Table 1. Summary of students' thinking processes

Ability category	Understanding the problem	Devising a plan	Carrying out the plan	Looking back
High	Complete; identifies known, asked, and inverse relation	Appropriate; selects inverse proportion	Accurate; uses $20 \times 30 = 50 \times x$ and obtains 12 days	Checks reasonableness independently
Moderate	Complete for known and asked information	Less appropriate; direct-proportion misconcepti	Inappropriate procedure; writes $20 : 50 = 30 : x$	Does not check independently

		on		
Low	Mostly understands known and asked information	Appropriate; recognizes inverse relation	Procedural/ computational difficulty	Does not check because of calculation uncertainty

Discussion

Relational Understanding as the Basis of Inverse Proportion

The findings show that students' main difficulty was not identifying numerical information but interpreting the relationship between quantities. All students recognized 20 workers, 30 days, and 50 workers, yet only S1 and S3 initially identified the inverse relationship. This indicates that word-problem solving requires students to construct a mathematical relation, not merely extract numbers. This finding supports Boonen et al. (2016), Daroczy et al., (2016), and Verschaffel et al., (2020), who emphasized that word problems require integration between comprehension and mathematical structure. S2's error occurred because the student treated the task as a routine direct proportion problem without considering that more workers should require fewer days. Therefore, inverse proportion should be taught through the invariant total work. The equation $20 \times 30 = 50 \times x$ represents a constant-product relation, not just a formula. Instruction should help students identify what remains constant and how each quantity changes.

Conceptual and Procedural Differences Across Ability Levels

The contrast among S1, S2, and S3 shows that conceptual and procedural knowledge appeared differently across ability levels. S1 demonstrated both conceptual understanding and procedural fluency by interpreting the context correctly, selecting the constant-product model, calculating accurately, and verifying the result. S2 understood the basic information but selected an inappropriate direct proportion relation by writing $20 : 50 = 30 : x$. This error shows a misconception in strategy selection because the student treated workers and time as quantities moving in the same direction. The finding is consistent with Rittle-Johnson, (2017) argument that conceptual and procedural knowledge are mutually supportive yet distinguishable, and with studies emphasizing schema recognition and relational interpretation in proportional reasoning (Jitendra et al. 2016; Smith et al. 2020). Meanwhile, S3 recognized the inverse relationship but struggled with multiplication and division, indicating that conceptual understanding requires procedural fluency to produce a valid solution.

Verification as a Marker of Reflective Problem Solving

The verification stage showed the clearest difference between S1 and the other students. S1 independently checked that 12 days was reasonable because the number of workers increased from 20 to 50, so the required time should decrease from 30 days. S2 and S3 did not verify their answers independently. S2 noticed the inconsistency only after the researcher prompted reconsideration of the worker-time relationship, while S3 avoided checking because of uncertainty in calculation. These findings indicate that verification is not yet an automatic habit for all students. Verification is closely related to metacognition, mathematical modelling, heuristic strategy use, and reflective problem solving (Hidayat et al. 2018; Mason and Singh 2016; Novotná et al. 2019). In this study, looking back functioned as metacognitive control.

Instructional Implications

The findings suggest that mathematics instruction should emphasize relational understanding before formula application. In inverse proportion, teachers can ask students to predict whether the answer should increase or decrease before calculating, so they first attend to the relationship between quantities. Symbolic forms should also be connected to contextual meaning; for example, $20 \times 30 = 50 \times x$ should be explained as a constant-work relationship, not merely a calculation rule. In addition, students need procedural fluency in multiplication, division, and equation solving so that correct conceptual recognition can lead to accurate solutions. Verification should also become a routine part of problem solving. Students can be guided to check whether the result is reasonable, whether the units make sense, and whether the answer fits the situation. Enriched word-problem experiences and digital prompts may support meaningful interpretation and reflective checking (Huang et al. 2016; Pongsakdi et al. 2020).

4. CONCLUSION

This study shows that students' thinking processes in solving inverse proportion problems vary by mathematical ability. The high-ability student understood the problem, selected the constant-product strategy, calculated $20 \times 30 = 50 \times x$, obtained 12 days, and verified the answer. The moderate-ability student understood the information but used an inappropriate direct proportion strategy. The low-ability student recognized the inverse relationship but struggled with multiplication and division and did not check the result. These findings indicate that errors may arise from relational misconceptions, procedural weakness, or limited verification. Instruction should emphasize conceptual relations, procedural fluency, and reflective checking.

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5. REFERENCES

Adeoye-Olatunde, O. A., and N. L. Olenik. 2021. "Research and Scholarly Methods: Semi-Structured Interviews." *Journal of the American College of Clinical Pharmacy* 4(10):1358–67. doi: 10.1002/jac5.1441.

Ayuningtyas, W., Mardiyana, and I. Pramudya. 2019. "Analysis of Students' Mathematical Problem Solving Skill Viewed from Mathematical Ability." *Journal of Physics: Conference Series* 1188:12089. doi: 10.1088/1742-6596/1188/1/012089.

Boonen, A. J. H., B. B. de Koning, J. Jolles, and M. van der Schoot. 2016. "Word Problem Solving in Contemporary Math Education: A Plea for Reading Comprehension Skills Training." *Frontiers in Psychology* 7:191. doi: 10.3389/fpsyg.2016.00191.

Boudreaux, A., S. E. Kanim, and S. W. Brahmia. 2016. "Student Facility with Ratio and Proportion: Mapping the Reasoning Space in Introductory Physics." *Physical Review Physics Education Research* 12(2):20102. doi: 10.1103/PhysRevPhysEducRes.12.020102.

Brahmia, S. W., A. Olsho, T. I. Smith, A. Boudreaux, P. Eaton, and C. Zimmerman. 2021. "The Physics Inventory of Quantitative Literacy: A Tool for Assessing Mathematical Reasoning in Introductory Physics." *Physical Review Physics Education Research* 17(2):20129. doi: 10.1103/PhysRevPhysEducRes.17.020129.

Cypress, B. S. 2017. "Rigor or Reliability and Validity in Qualitative Research: Perspectives, Strategies, Reconceptualization, and Recommendations." *Dimensions of Critical Care Nursing* 36(4):253–63. doi: 10.1097/DCC.0000000000000253.

Daroczy, G., M. Wolska, W. D. Meurers, and H. C. Nuerk. 2016. "Word Problems: A Review of Linguistic and Numerical Factors Contributing to Their Difficulty." *Frontiers in Psychology* 7:348. doi: 10.3389/fpsyg.2016.00348.

Fuchs, L. S., D. Fuchs, A. S. Malone, A. Y. Wang, C. L. Hamlett, and S. Krowka. 2021. "Effects of Intervention to Improve At-Risk Fourth Graders' Understanding, Calculations, and Word Problems with Fractions." *Exceptional Children* 87(3):290–308. doi: 10.1177/0014402920985226.

Hidayat, R., H. Zulnaidi, and S. N. A. S. Zamri. 2018. "Roles of Metacognition and Achievement Goals in Mathematical Modeling Competency: A Structural Equation Modeling Analysis." *PLOS ONE* 13(11):e0206211. doi: 10.1371/journal.pone.0206211.

Huang, X., S. D. Craig, J. Xie, A. Graesser, and X. Hu. 2016. "Intelligent Tutoring Systems Work as a Math Gap Reducer in 6th Grade After-School Program." *Learning and Individual Differences* 47:258–65. doi: 10.1016/j.lindif.2016.01.012.

Jitendra, A. K., D. N. Dupuis, J. R. Star, and M. C. Rodriguez. 2016. "The Effects of Schema-Based Instruction on the Proportional Thinking of Students with Mathematics Difficulties." *Journal of Learning Disabilities* 49(4):354–67. doi: 10.1177/0022219414554228.

Johnson, J. L., D. Adkins, and S. Chauvin. 2020. "A Review of the Quality Indicators of Rigor in Qualitative Research." *American Journal of Pharmaceutical Education* 84(1):7120. doi: 10.5688/ajpe7120.

Krawitz, J., S. Schukajlow, and W. Van Dooren. 2018. "Unrealistic Responses to Realistic Problems with Missing Information: What Are Important Barriers?" *Educational Psychology* 38(10):1221–38. doi: 10.1080/01443410.2018.1502413.

Levitt, H. M., M. Bamberg, J. W. Creswell, D. M. Frost, R. Josselson, and C. Suárez-Orozco. 2018. "Journal Article Reporting Standards for Qualitative Primary, Qualitative Meta-Analytic, and Mixed Methods Research in Psychology: The APA Publications and Communications Board Task Force Report." *American Psychologist* 73(1):26–46. doi: 10.1037/amp0000151.

Liljedahl, P., M. Santos-Trigo, U. Malaspina, and R. Bruder. 2016. *Problem Solving in Mathematics Education*. Springer.

Mason, A., and C. Singh. 2016. "Using Reflection with Peers to Help Students Learn Effective Problem Solving Strategies." *Physical Review Physics Education Research* 12(1):10103. doi: 10.1103/PhysRevPhysEducRes.12.010103.

Novotná, J., P. Eisenmann, J. Přibyl, J. Ondrušová, and J. Břehovský. 2019. "Problem Solving in School Mathematics Based on Heuristic Strategies." *Journal on Efficiency and Responsibility in Education and Science* 12(1):1–6. doi: 10.7160/eriesj.2019.120101.

Phillips, J. A., J. E. McCallum, K. W. Clemmer, and T. M. Zachariah. 2016. "A Problem-Solving Framework to Assist Students and Teachers in STEM Courses." *Journal of College Science Teaching* 46(2):33–39. doi: 10.2505/4/jcst16_046_02_33.

Pongsakdi, N., T. H. Laine, K. Veermans, M. M. Hannula-Sormunen, and E. Lehtinen. 2020. "Improving Word Problem Performance in Elementary School Students by Enriching Word Problems Used in Mathematics Teaching." *LUMAT: International Journal on Math, Science and Technology Education* 8(1):23–44. doi: 10.31129/LUMAT.8.1.1318.

Rittle-Johnson, B. 2017. "Developing Mathematics Knowledge." *Child Development Perspectives* 11(3):184–90. doi: 10.1111/cdep.12229.

Rott, B., B. Specht, and C. Knipping. 2021. "A Descriptive Phase Model of Problem-Solving Processes." *ZDM Mathematics Education* 53:737–52. doi: 10.1007/s11858-021-01244-2.

Schukajlow, S., G. Kaiser, and G. Stillman. 2018. "Empirical Research on Teaching and Learning of Mathematical Modelling: A Survey on the Current State-of-the-Art." *ZDM Mathematics Education* 50:5–18. doi: 10.1007/s11858-018-0933-5.

Smith, T. I., S. W. Brahmia, A. Olsho, A. Boudreaux, P. Eaton, P. J. Kelly, K. J. Louis, M. A. Nussenbaum, and L. J. Remy. 2020. "Developing a Reasoning Inventory for Measuring Physics Quantitative Literacy." *Physical Review Physics Education Research* 16(2):20123. doi: 10.1103/PhysRevPhysEducRes.16.020123.

Verschaffel, L., S. Schukajlow, J. Star, and W. Van Dooren. 2020. "Word Problems in Mathematics Education: A Survey." *ZDM Mathematics Education* 52:1–16. doi: 10.1007/s11858-020-01130-4.